MATH 153 Practice Final Exam

Do as many questions as you can. Every question you do counts. The maximum number of points you may earn is 100. The number next to a question is the maximum score allocated to the question. In the exam itself, you will be requred to write down the steps leading to your answers. If you do not, you will earn no ponts for a solution that is not defended by any work.

1. Determine the derivative of the given function and simplify when possible.

   (a) $f(x) = \frac{4}{5}x^{10} - \frac{1}{3x^3} - \frac{x}{4} + 8$ (4)

   (b) $g(x) = 3\cos x + 5\csc x - 5\cot x + 12\tan x - \ln x$ (No need to show work for this.) (5)

   (c) $h(x) = 7x\cos x - 9e^x \sin x$ (4)

   (d) $u(x) = \frac{3x}{4x - 5}$ (4)

   (e) $v(x) = \frac{\sin x}{2 + \cos x}$ (4)

   (f) $w(x) = 5x\left(\sqrt{x^2 + 4}\right) - \tan 5x$ (6)

   (g) $f(x) = \sec^2(4x) - 2\cot(4x) + 15$ (5)

   (h) $v(x) = 3 - \arcsin\left(\frac{1}{3}x\right) + \arccos\left(\frac{1}{2}x\right)$ (4)

   (i) $h(x) = 1 - 4\left(\cos\frac{1}{2}x\right)\left(e^{2x}\right)\ln\left(x^2 + 1\right)$ (6)

2. You are given $f(x) = e^{3x}$.

   (a) What is $f'(x)$? (2)

   (b) What is $f''(x)$? (2)

   (c) What is $f'''(x)$? (2)

   (d) What is your guess for $f^{(10)}(x)$, the tenth derivative of f? (2)

   (e) What is your guess for $f^{(n)}(x)$, the nth derivative of f? (2)

3. Find the derivative of the function $g(x) = (2x)^{x^2}$ (6)

4. Determine $\lim\limits_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$. (6)

5. Calculate the derivative of $g(x) = \arctan\left(\frac{1}{x^2}\right)$ then show that it simplifies to $g'(x) = \frac{-2x}{x^4 + 1}$ (6)

6. A function y is defined implicitly by $x^2 + xy - y^2 = -1$

   (a) Find $\frac{dy}{dx}$, (you must show your work.) (4)

   (b) Determine the slope of the tangent to the graph of y at $(1, -1)$. (2)

   (c) Determine the equation of the tangent to the graph of y at $(1, -1)$. (3)

7. The volume V of a sphere is increasing at the rate of 12 cubic centimeters per second. At what rate is its radius r changing when it, (i.e. the radius r), is 8 centimeters? [The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] (5)

8. Find the two critical points of $f(x) = 3x^2 - x^3$ and establish their nature. (6)

9. The graph below of the function $f(x) = x^3 - 2x - 3$ suggests that $x_0 = 2$ is an approximate solution of the equation $x^3 - 2x - 3 = 0$
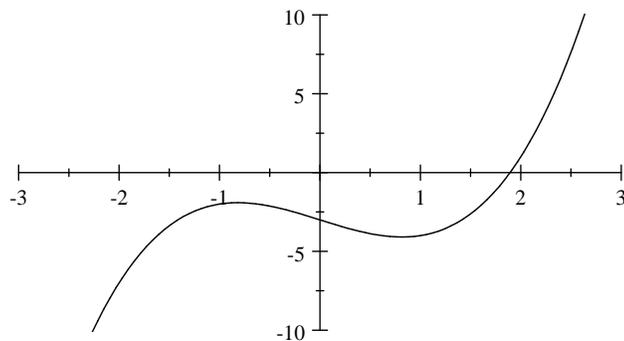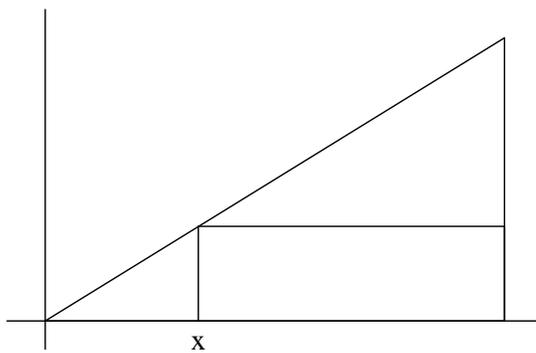

Use Newton's method to obtain better approximate solution of the equation which is accurate to two decimal places. (7)

10. A triangle has vertices at $P(0,0)$, $Q(6,0)$ and $R(6,5)$ and a rectangle is drawn inside the triangle as shown in the diagram.
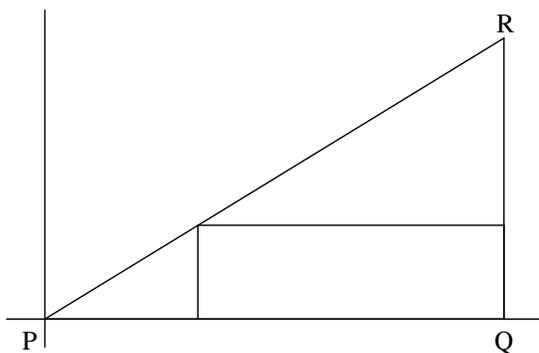
(a)

(b)

(a) Show that the equation of the line segment PR is $y = \frac{5}{6}x$. (2)

(b) Show that when the left vertical side of the rectangle intersects the horizontal axis at $(x, 0)$ as shown in figure (b) then the area of the rectangle is $\frac{5}{6}x(6-x)$ square units. (2)

(c) What value of x gives the largest value of the rectangle's area. (3)

11. You are given the function $f(x) = \sqrt{5+4x}$ and the point $c = 1$. Determine the 3rd degree Taylor polynomial for f at the point $c = 1$. (6)