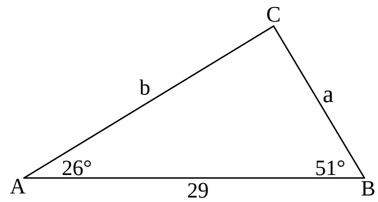
Solutions tosome of the practice problems

1. To solve the triangle ABC with $A = 26°$, $B = 51°$ and $c = 29$ cm.: It is useful to draw a figure that shows the given information.



To get angle C, use the fact that the sum of the angles of a triangl is 180°. Therefore $C = 180° - 26° - 51° = 103°$. Now we know $\dfrac{c}{\sin C}$ and it is equal to $\dfrac{29}{\sin 103°}$. We use it, and the law of sines, to calculate the lengths a and b.

$$\frac{a}{\sin 26°} = \frac{29}{\sin 103°}, \quad \text{therefore} \quad a = \frac{29 \sin 26°}{\sin 103°} = 13.0, \ \text{rounded to 1 decimal place.}$$

$$\frac{b}{\sin 51°} = \frac{29}{\sin 103°}, \quad \text{therefore} \quad b = \frac{29 \sin 51°}{\sin 103°} = 23.1, \ \text{rounded to 1 decimal place.}$$

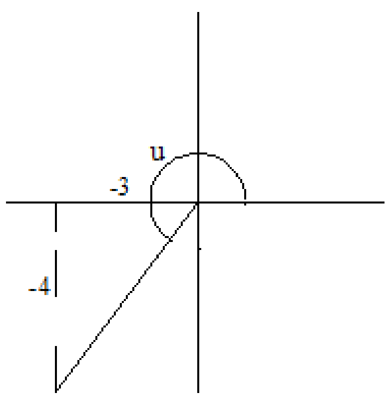
2. Convert the angle 105° into radians and express your answer as a multiple of π.

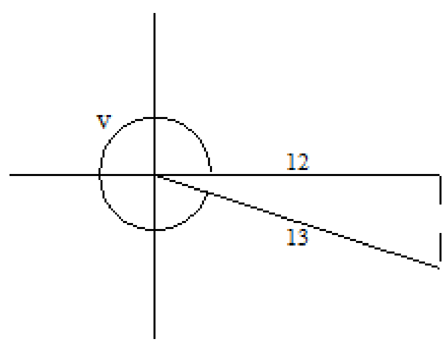
$$105° = \frac{\pi \times 105}{180} \ \text{radians. This may be simplified to} \ \frac{7\pi}{12} \ \text{radians}$$

3. Find the area, to the nearest square inch, of the triangle ABC with $A = 21°$, $b = 10$ inches and $c = 8$ inches. This is a SAS triangle and its area is

$$\tfrac{1}{2} bc \sin A = \tfrac{1}{2} \times 10 \times 8 \times \sin 21° = 14.3 \ \text{square inches, rounded to 1 decimal place.}$$

4. You are given that u is an angle in quadrant III with $\tan u = \frac{4}{3}$ and that v is an angle in the fourth quadrant with $\cos v = \frac{12}{13}$. Draw the angles then calculate the exact value of $\sin(v - u)$.





In the triangle formed from angle u, the hypotenuse is unknown. If it is h then

$$h^2 = 3^2 + 4^2 = 25 \quad \text{therefore} \quad h = 5. \text{ It follows that } \sin u = -\tfrac{4}{5} \ \text{and} \ \cos u = -\tfrac{3}{5}$$

In the triangle formed from angle v, one of the shorter sides is unknown. If it is a then

$13^2 = 12^2 + a^2$ therefore $a^2 = 25$ and so $a = \pm 5$. We take $a = -5$. It follows that $\sin v = -\frac{5}{13}$

$$\sin(v - u) = \sin u \cos v - \cos u \sin v = \left(-\tfrac{4}{5}\right)\left(\tfrac{12}{13}\right) - \left(-\tfrac{3}{5}\right)\left(-\tfrac{5}{13}\right) = -\frac{48}{65} - \frac{15}{65} = -\frac{63}{65}$$

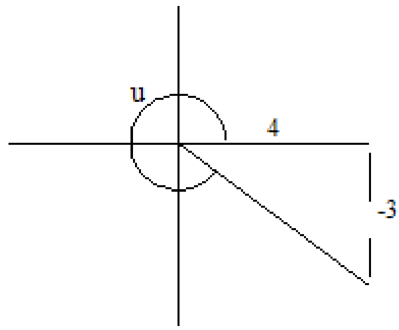
5. To find the exact value of $\sin 190° \cos 70° - \cos 190° \sin 70°$. We use the formula

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

in reverse order with $A = 190°$ and $B = 70°$. The result is

$$\sin 190° \cos 70° - \cos 190° \sin 70° = \sin(190° - 70°) = \sin 120° = \tfrac{\sqrt{3}}{2}$$

6. You are given that u is an angle in standard position and that the point $(4, -3)$ is on its terminal ray. Draw the angle then determine the exact value of $\sin u$.



The unknown side in the right-triangle given by the angle u is the hypotenuse. If its length is h then

$$h^2 = 4^2 + 3^2 = 25, \text{ therefore } h = 5. \quad \text{It follows that} \quad \sin u = -\tfrac{3}{5}$$

7. To convert the angle $\frac{59}{18}\pi$ radians into degrees.

$$\tfrac{59}{18}\pi \text{ radians} = \tfrac{59}{18}\pi \times \tfrac{180}{\pi} \text{ degrees. This is equal to } \tfrac{59}{18} \times \tfrac{180}{1} = 590 \text{ degrees}$$

8. To write $\cos 7u - \cos 3u$ as a product of two trigonometric functions.

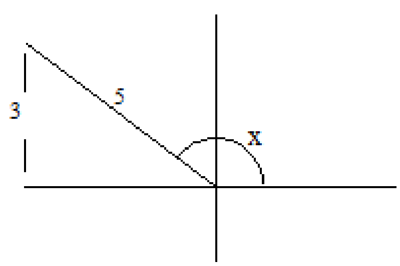
$$\cos 7u - \cos 3u = -2\left[\sin \tfrac{1}{2}(7u + 3u) \sin \tfrac{1}{2}(7u - 3u)\right] = -2 \sin 5u \sin 2u$$

9. A gear with a radius of 16 centimeters is turning at $\dfrac{\pi}{9}$ radians per second. What is the linear speed of a point on the outer edge of the gear in centimeters per second?

Since the angles are measured in radians, Linear speed = Angular speed × Radius of gear

$$\text{Therefore linear speed of point on the rim of the gear is } \frac{\pi}{9} \times 16 = \frac{16\pi}{9} \text{ cm per sec.}$$

10. Given that $\sin x = \frac{3}{5}$ and $\cos x < 0$, to calculate the exact value of $\cos \frac{x}{2}$. Since $\sin x$ is positive and $\cos x$ is negative, x is an angle in the second quadrant.



11. To use a sum or difference identity and the given table for values of trigonometric functions to find the exact value of $\cos(135° + 120°)$.

$$\cos(135° + 120°) = \cos 135° \cos 120° - \sin 135° \sin 120° = \left(-\tfrac{\sqrt{2}}{2}\right)\left(-\tfrac{1}{2}\right) - \left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

12. To write the product $\sin 6u \sin 2u$ as a sum or difference of two trigonometric functions.

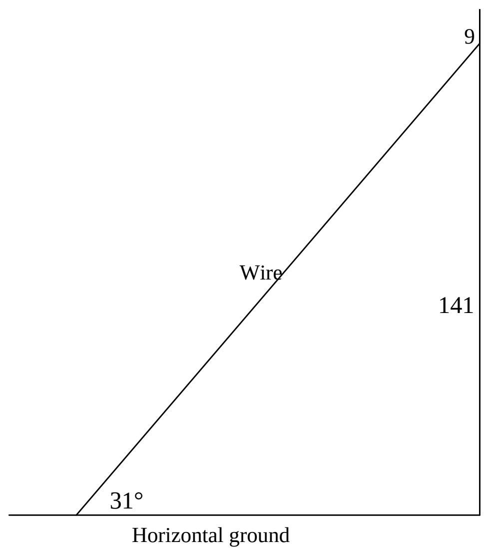
$$\sin 6u \sin 2u = \frac{1}{2}\left[\cos(6u - 2u) - \cos(6u + 2u)\right] = \frac{1}{2}\left[\cos 8u - \cos 4u\right]$$

13. To find the area of the triangle ABC with $a = 18$ yards, $b = 12$ yards and $c = 14$ yards, using Heron's formula, and round off to the nearest square yard. For this triangle,

$$s = \frac{18 + 12 + 14}{2} = 22, \quad s - a = 4, \quad s - b = 10, \quad \text{and} \quad s - c = 8$$

Therefore the area of the triangle is $\sqrt{s\,(s-a)\,(s-b)\,(s-c)} = \sqrt{22 \times 4 \times 10 \times 8} = \sqrt{7040} = 84$ square yards, (to the nearest square yard).

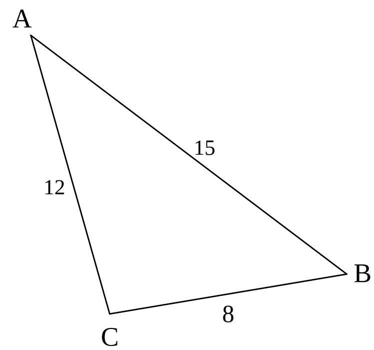
14. A radio transmission tower is 150 feet tall. How long should a guy wire be if it is to be attached 9 feet from the top of the tower and is to make an angle of 31° with the horizontal ground. Round your answer to the nearest tenth of a foot.



From the figure, if the length of the wire is h then

$$\frac{141}{h} = \sin 31^\circ \quad \text{Therefore} \quad h = \frac{141}{\sin 31^\circ} = 273.8 \ \text{feet, rounded to the nearest tenth of a foot.}$$

15. Draw a big figure of the triangle ABC with $a = 8$ cm, $b = 12$ cm and $c = 15$ cm then solve it.



Using the law of cosines,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{12^2 + 15^2 - 8^2}{2 \times 12 \times 15} = 0.84722.$$

Therefore $A = \cos^{-1}(0.84722) = 32°$, rounded to the nearest degree. Applying the same law, gives

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{8^2 + 15^2 - 12^2}{2 \times 8 \times 15} = 0.60417$$

Therefore $B = \cos^{-1}(0.60417) = 53°$, rounded to the nearest degree. We may calculate the remaining angle by subtraction the above two angles from 180°. Thus $C = 180° - 32° - 53° = 95°$.

16. To find the period and phase shift of the trigonometric function $f(x) = 7\sin(4x - 80°)$. We write it as

$$f(x) = 7\sin 4(x - 20°)$$

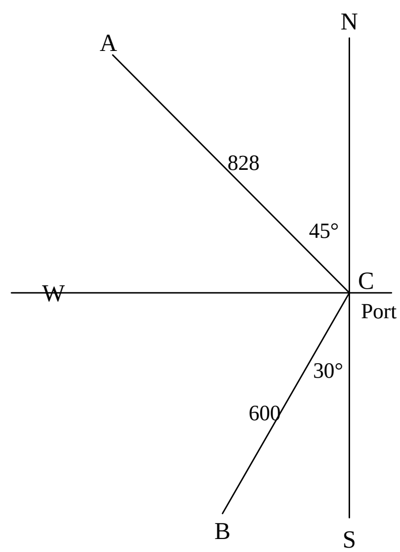
The period is $\dfrac{360°}{4} = 90°$. The phase shift is 20° to the right.

17. A car wheel has a 14-inch radius. To determine the angle (to the nearest degree) that the wheel turns when the car rolls forward 1 foot.

When the car moves 1 foot, which is equal to 12 inches, an arc on the tire which is 12 inches long comes in contact with the ground. The angle u that subtends an arc of length 12 inches in a circle of radius 14 inches is obtained by solving the equation

$\dfrac{\pi \times 14 \times u}{180} = 12$. The result is $u = \dfrac{12 \times 180}{14 \times \pi} = 49°$, rounded to the nearest degree.

18. Two airplanes leave an airport at 12:00 noon, one flying on bearing $N45°W$ at 414 miles per hour and the other one flying on bearing $S30°W$ at a speed of 300 miles per hour. To calculate : (a) The distance of the faster plane from the airport at 2:00 pm, (b) The distance of the slower plane from the airport at 2:00 pm, and the distance between the two planes at 2:00 pm.

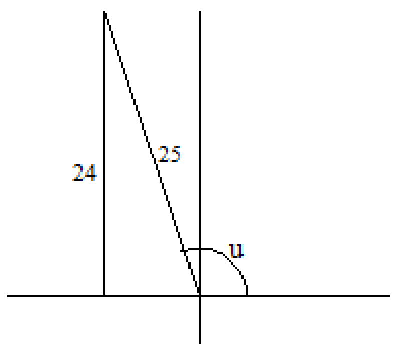


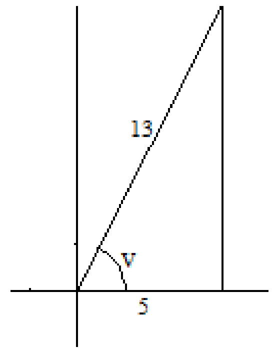
At 2:00 pm, which is two hours sincethe planes took off, the faster plane is 828 miles and the slower one is 600 miles from the airport C as shown on the figure above.The distance between the two planes at 2:00 pm is the length of side AB of triangle ABC. Angle C is $45° + 70° = 115°$. Therefore the distance c between the planes can be calculated using the law of cosines and it is given by

$$c^2 = 828^2 + 600^2 - 2(828)(600)\cos 115° = 1465497.505$$

Therefore $c = \sqrt{1465497.505} = 1210.57$. To the nearest mile, the plane are 1210 miles apart at 2:00 pm

19. You are given that u is an angle in quadrant II with $\sin u = \frac{24}{25}$ and that v is an angle in the first quadrant with $\cos v = \frac{5}{13}$. Draw the angles then calculate the exact value of $\cos(u - v)$.





In the right-triangle for angle u, if the unknown side has length a then

$a^2 + 24^2 = 25^2$, therefore $a^2 = 49$. Taking square roots gives $a = \pm 7$. Take $a = -7$ because horizontal coordinates are negative in the second quadrant. Therefore $\cos u = -\frac{7}{25}$

In the right-triangle for angle v, if the unknown side has length b then

$b^2 + 5^2 = 13^2$, therefore $b^2 = 144$. Taking square roots gives $b = \pm 12$. Take $b = 12$ because vertical coordinates are positive in the first quadrant. Therefore $\sin v = \frac{12}{13}$.

$$\cos(u - v) = \cos u \cos v + \sin u \sin v = \left(-\tfrac{7}{25}\right)\left(\tfrac{5}{13}\right) + \left(\tfrac{24}{25}\right)\left(\tfrac{12}{13}\right) = \frac{-35 + 12 \times 24}{25 \times 13} = \frac{254}{325}$$

20. Find the length of the arc subtended by an angle of $35°$ on a circle of radius 55 inches.

$$\text{The arclength is } \frac{\pi\,(55)\,(35)}{180} = 33.6 \text{ inches, (rounded to 1 decimal place).}$$

21. Write $\sin 8u - \sin 2u$ as a product of two trigonometric functions.

$$\sin 8u - \sin 2u = 2\left[\cos \tfrac{1}{2}(8u + 2u) \sin \tfrac{1}{2}(8u - 2u)\right] = 2 \cos 5u \sin 3u$$

22. Find all the solutions of the equation $2 \sin x + 1 = 0$.

If $2 \sin x + 1 = 0$ then $\sin x = -\frac{1}{2}$, thus x is an angle whose sine is $-\frac{1}{2}$. I.e. $x = \sin^{-1}\left(-\frac{1}{2}\right) = -30°$

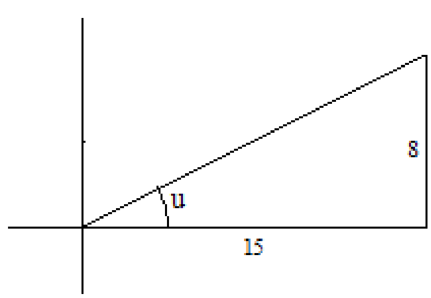
Another angle is $x = 210°$. Since we may add any integer multiple of $360°$ to any one of the above two angles, the solutions are

$$x = (-30 + 360n)° \quad \text{or} \quad x = (210 + 360n)° \quad \text{where } n \text{ is any integer.}$$

23. You are given that u is an angle in standard position and $\sin u > 0$ but $\cos u < 0$. In what quadrant is u?

Since $\sin u$ is positive, u is in the first or second quadrant. It is alsogiven that $\cos u$ is negative, thus u is in the second or third quadrant. To satisfy both conditions, u must be in the second quadrant

24. $\tan^{-1}\left(\frac{8}{15}\right)$ is an angle u in the first quadrant with $\tan u = \frac{8}{15}$. Draw it then calculate the exact value of $\cos\left(\tan^{-1}\left(\frac{8}{15}\right)\right)$.



In the right-angled triangle for u, the unknown side is the hypotenuse. It its length is h then

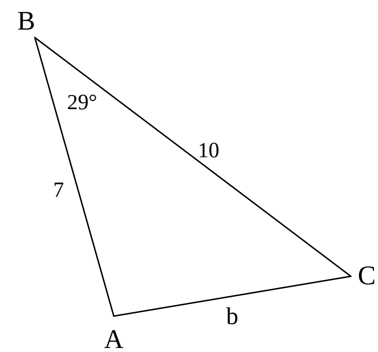
$$h^2 = 8^2 + 15^2 = 289. \text{ Therefore } h = \sqrt{289} = 17$$

It follows that $\cos u = \frac{15}{17}$. In other words, $\cos\left(\tan^{-1}\left(\frac{8}{15}\right)\right) = \frac{15}{17}$

25. Draw a diagram showing an angle of 397° then determine its reference angle. Highlight the reference angle on your diagram.

The reference angle for 397° is 37°

26. Draw a big figure of triangle ABC with $a = 10$ cm, $B = 29°$ and $c = 7$ cm then solve it.



We use the law of cosines to determine b:

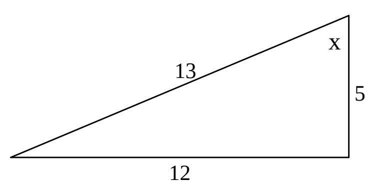
$$b^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \cos(29°) = 26.55$$

Therefore $b = \sqrt{26.55} = 5.2$ cm. (rounded to 1 decimal place). We have to calculate one of the angles using the law of cosines or the law of sines. Using the law of cosines

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{26.55 + 49 - 100}{2 \times 5.2 \times 10} = -0.2351$$

Therefore $A = \cos^{-1}(-0.2351) = 104°$, rounded to the nearest degree

27. Use the right angled triangle below to find $\sin x$ and $\cos x$ then determine the exact value of $\sin 2x$.



From the figure, $\sin x = \frac{12}{13}$ and $\cos x = \frac{5}{13}$. Therefore

$$\sin 2x = 2 \sin x \cos x = \left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{60}{169}$$

28. A painter has to paint a triangular region which is 65 meters by 67 meters by 74 meters.

(a) Use Heron's formula to calculate the area of the region, to the nearest square meter.

We determine $s = \dfrac{65 + 67 + 74}{2} = 103, \quad s - 65 = 38, \quad s - 67 = 36, \quad \text{and} \quad s - 74 = 29$

The area of the triangle is $\sqrt{s(s-65)(s-67)(s-74)} = \sqrt{103 \times 38 \times 36 \times 29} = 2021.4$ square meters, rounded to 1 decimal place.

(b) If one full can of paint covers 70 square meters, how many full cans should he bring along?

Since $2021.4 \div 70 = 28.9$, he has to bring 29 full cans

29. Write $\sin 3w + \sin 5w$ as a product of two trigonometric functions.

$$\sin 3w + \sin 5w = 2\left[\sin \tfrac{1}{2}(3w + 5w)\cos \tfrac{1}{2}(3w - 5w)\right] = 2\sin 4w\cos(-w) = 2\sin 4w\cos w$$

30. Find all the solutions of the equation $2\sin x - \sqrt{3} = 0$.

If $2\sin x - \sqrt{3} = 0$ then $\sin x = \frac{\sqrt{3}}{2}$, thus x is an angle whose sine is $\frac{\sqrt{3}}{2}$. I.e. $x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$

Another angle is $x = 120°$. Since we may add any integer multiple of $360°$ to any one of the above two angles, the solutions are

$$x = (60 + 360n)^\circ \quad \text{or} \quad x = (120 + 360n)^\circ \quad \text{where } n \text{ is any integer.}$$

31. A truck has tires with diameter 24 inches and it is travelling at 32 miles per hour.

(a) Determine its speed in inches per minute and round off your answer to 1 decimal place. (There are 63360 inches in 1 mile.)

Speed in inches per minute is $32 \times 63360 = 2027520$

(b) Determine the angular speed of its tires in radians per minute and round off your answer to 1 decimal place.

If the angular speed is w radians per minute then $12w = 2027520$, therefore $w = \dfrac{2027520}{12} = 168960$.

(c) How many revolutions do its tires turn per minute? Round off your answer to the nearest whole number.

There are 2π radians in one revolution. Therefore the tires turn $\dfrac{168960}{2\pi} = 26891$ revolutions per minute

32. Two sailboats leave a harbor in the Bahamas at the same time. The first one sails at 25 miles per hour on a bearing $S70°E$. The second one sails at 32 miles per hour on a bearing $N40°E$. Assume that they maintain their speeds and bearings for 3 hours.

(a) How far is the first boat from the port three hours later?

(b) How far is the second boat from the port three hours later?

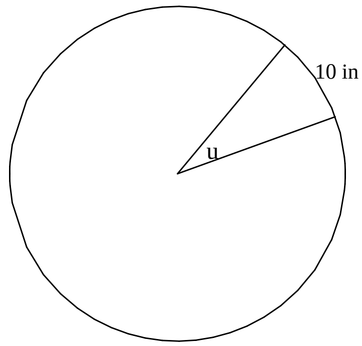
(c) How far apart, to the nearest mile, are the two boats three hours later?

Do this question the same way Question 18 was done. Three hours later, the first boat was 75 miles from the harbor. The second one was 96 miles from the harbor. If the distance between the boats three hours later was c then c^2 is equal to $75^2 + 96^2 - 2 \times 75 \times 96\cos 70°$, therefore c rounds to 100 miles.

33. Convert an angular speed of 270 revolutions per minute into an angular speed in radians per second.

Since 1 revolution is equal to 2π radians, 270 revolutions equal $2\pi \times 270$ radians. To convert the angular speed in radians per second, we divide the angular speed in radians per minute, by 60 because 60 seconds equal i minute. Therefore angulat speed is $\dfrac{2\pi \times 270}{60}$ radians per second. This rounds up to 28.3 radians per second.

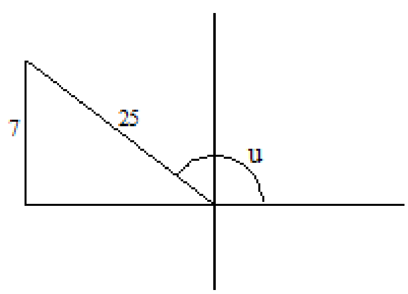
34. A wheel with a 20-inch radius is marked at two points on the rim. The distance between the marks along the wheel is found to be 10 inches. What is the angle (to the nearest tenth of a degree) between the radii to the two marks?

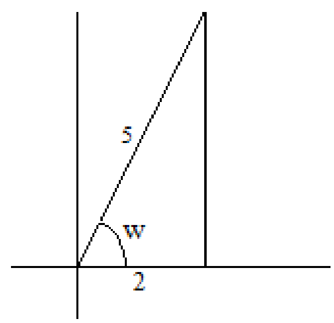


The wheel may be viewed as a circle with radius 20 inches. The length of the arc connecting the two points is 10 inches. If angle between the two radii is u then

$$\frac{\pi \times 20 \times u}{180} = 10. \text{ Therefore } u = \frac{180 \times 10}{20\pi} = 28.6^\circ, \text{ (to the nearest tenth of a degree).}$$

35. You are given that u is an angle in the second quadrant with $\sin u = \frac{7}{25}$ and that w is an angle in the first quadrant with $\cos w = \frac{2}{5}$. Draw the two angles then calculate the exact value of $\sin(u - w)$.





The unknown side of the right angled triangle given by angle u has length a given by

$a^2 + 7^2 = 25^2$. Therefore $a^2 = 576$ and so $a = \pm 24$. Take $a = -24$, (horiz. coord. are negative in Quadrant II)

The unknown side of the right angled triangle given by angle w has length b given by

$b^2 + 2^2 = 5^2$. Therefore $b^2 = 21$ and so $b = \pm\sqrt{21}$. Take $b = \sqrt{21}$, (vert. coord. are positive in Quadrant I)

Since $\cos u = -\frac{24}{25}$ and $\sin w = \frac{\sqrt{21}}{5}$, it follows that

$$\sin(u - w) \sin u \cos w - \cos u \sin w = \left(\frac{7}{25}\right)\left(\frac{2}{5}\right) - \left(-\frac{24}{25}\right)\left(\frac{\sqrt{21}}{5}\right) = \frac{14 + 24\sqrt{21}}{125}$$

36. Verify the identity $\csc x (\sin x + \cos x) = 1 + \cot x$.

The Left Hand Side is $\csc x (\sin x + \cos x) = \frac{1}{\sin x}(\sin x + \cos x) = 1 + \frac{\cos x}{\sin x} = 1 + \cot x$, which is the right hand side. Therefore the identity is verified.