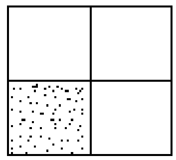
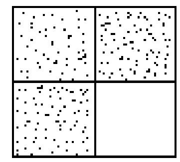
Review of Fractions

We are introduced to fractions as "parts of a whole". For example, $\frac{1}{4}$ of a rectangle is one of four pieces you get when you cut the rectangle into four equal pieces; and $\frac{3}{4}$ of the rectangle is 3 of the four equal pieces.



$\frac{1}{4}$ of a rectangle is shaded



$\frac{3}{4}$ of a rectangle is shaded

In general, if n is a positive integer then $\frac{1}{n}$ of a rectangle is **one** of the n pieces you get when you divide the rectangle into n **equal pieces**. Naturally, $\frac{2}{n}$ is **two** of the n equal pieces, etc. In general, $\frac{m}{n}$ means m of these n equal pieces. In case you have forgotten n is called the denominator, and m the numerator, of $\frac{m}{n}$. Also, the fractions $\frac{m}{n}$ where m and n are whole numbers and n is not zero, are called *rational number*.

Exercise 1 *Shade $\frac{3}{7}$ of the rectangle in figure (i) and $\frac{5}{6}$ of the rectangle in figure (ii).*

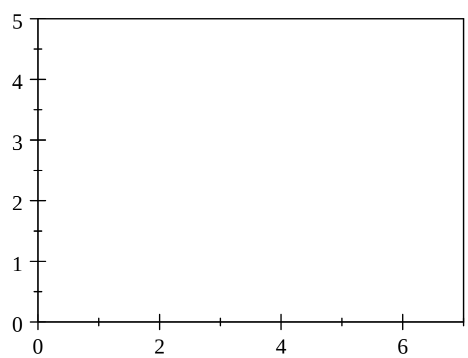


Figure (i)

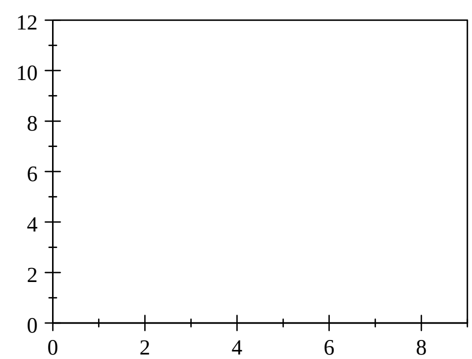


Figure (ii)

In general, it is meaningful to consider "$\frac{1}{2}$ of a class", "$\frac{2}{3}$ of a given bill", "$\frac{3}{4}$ of the voters in a certain county", etc. When we say that "$\frac{3}{5}$ of a class of 40 students" own a smart phone, we mean that the class can be divided into five equal groups, (equal numerically), in such a way that each of the students in three of these groups owns a smart phone. Clearly, each of the five groups contains 8 students, therefore a total of 24 students, (i.e. 3×8 students), in the class own a smart phone. Thus

$$\frac{3}{5} \text{ of } 40 \text{ means } \frac{40}{5} \times 3 = 24$$

Example 2 *$\frac{2}{3}$ of the gas bill one pays at a pump in some country is tax. How much of a \$48.60 gas bill is tax?*

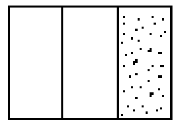
Solution: *$\frac{2}{3}$ of 48.60 means $\frac{48.60}{3} \times 2 = 16.20 \times 2 = 32.40$. Therefore \$32.40 of the \$48.60 is tax.*

Exercise 3

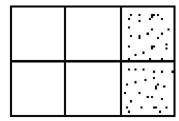
1. *$\frac{3}{4}$ of the 84000 voters in a certain county voted . How many of the 84000 voted?*
2. *$\frac{3}{8}$ of the 7400 students in a certain college have scholarships. How many of the 7400* ***do not*** *have scholarships?*

All the fractions in the above examples have numerators that are smaller than the corresponding denominators, but this does not have to be the case. For example it is possible to have a dollar and one quarter, which we may write as $1\frac{1}{4}$ dollars. If one views one dollar as 4 quarters then $1\frac{1}{4}$ dollars may be viewed as five quarters, which we write as $\frac{5}{4}$.

Fractions that represent "the same amount" are called equivalent fractions. For example, $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent.

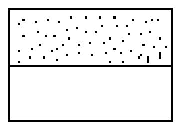


$\frac{1}{3}$

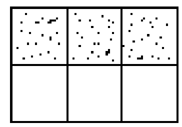


$\frac{2}{6}$

Another pair of equivalent fractions is $\frac{1}{2}$ and $\frac{3}{6}$



$\frac{1}{2}$



$\frac{3}{6}$

An easy way of obtaining equivalent fractions is *to multiply or divide the numerator and denominator of a given fraction by the* ***same non-zero*** *number.* For example given the fraction $\dfrac{20}{50}$:

- Multiplying its numerator and denominator by 15 gives the equivalent fraction

$$\frac{20 \times 15}{50 \times 15} = \frac{300}{750}$$

- Dividing its numerator and denominator by 5 gives the equivalent fraction

$$\frac{20 \div 5}{50 \div 5} = \frac{4}{10}.$$

- Dividing its numerator and denominator by -10 gives the equivalent fraction

$$\frac{20 \div (-10)}{50 \div (-10)} = \frac{-2}{-5}$$

- Multiplying the numerator and denominator by 0.8 gives the equivalent fraction

$$\frac{20 \times 0.8}{50 \times 0.8} = \frac{16}{40}$$

Dividing the numerator and denominator of a given fraction by the same number is a common way of simplifying fractions. For example, given the fraction $\dfrac{36}{48}$ we may write it as $\dfrac{3 \times 12}{4 \times 12}$ then divide the numerator and denominator by 12, (also called cancelling 12), to get

$$\frac{36}{48} = \frac{3 \times 12}{4 \times 12} = \frac{3 \times 12 \div 12}{4 \times 12 \div 12} = \frac{3}{4}.$$

This course will also involve fractions with variable numerators or denominators. Examples:

$$\frac{3}{n}, \frac{4}{m+2}, \frac{y}{5}, \frac{n+12}{6}.$$

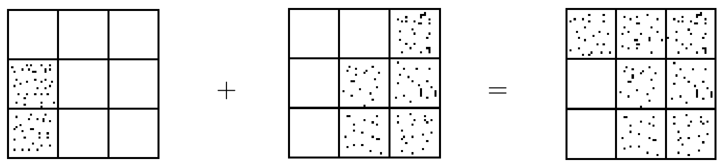
They are not any different from the fractions with specific numbers in the numerators and denominators. In particular, equivalent fractions are obtained by multiplying/dividing the numerators and denominators by the same number. Examples:

- $\frac{3}{n}, \frac{9}{3n}, \frac{24}{8n}, \frac{3y}{ny}$ are all equivalent fractions.
- $\frac{4}{m+2}, \frac{12}{3(m+2)}, \frac{72}{18(m+2)}, \frac{4x}{x(m+2)}$ are also equivalent fractions.

Note that $\frac{4}{m+2}$ and $\frac{12}{m+6}$ are not equivalent. The reason is that, although we multiplied the numerator of $\frac{4}{m+2}$ by 3 to get 12, we did not multiply the denominator $m+2$ by 3. We only multiplied the 2 in the denominator by 3 to get 6. We should have multiplied the whole expression $m+2$ by 3 to get $3m+6$. Thus $\frac{4}{m+2}$ and $\frac{12}{3m+6}$ are equivalent.

Addition/Subtraction

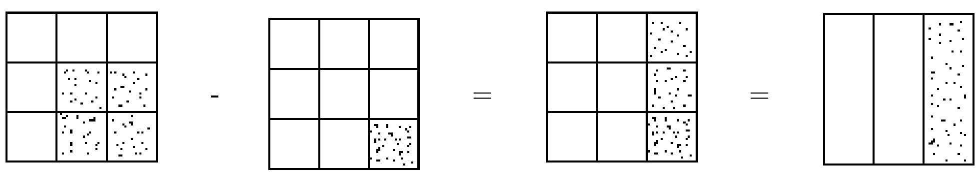
Fractions that have the same denominators are called "like fractions". It is easy to add such a pair numbers. For example, the sum of $\frac{2}{9}$ and $\frac{5}{9}$ is $\frac{7}{9}$. This is illustrated below using rectangles.



Subtraction is equally simple. For example

$$\frac{4}{9} - \frac{1}{9} = \frac{4-1}{9} = \frac{3}{9} = \frac{1}{3}$$

Using rectangles:



Before you add or subtract fractions that have different denominators, you have to replace them with equivalent fractions that have the same denominator then add/subtract as we did above. For example, to add the two fractions $\frac{1}{6}$ and $\frac{3}{4}$, you must first get a common denominator. This should be a number that is divisible by 6 ad also by 4. A possible candidate is 12, therefore replace $\frac{1}{6}$ with the identical fraction $\frac{2}{12}$ and replace $\frac{3}{4}$ with the identical fraction $\frac{9}{12}$. Now you may add:

$$\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{2+9}{12} = \frac{11}{12}$$

In general, given a pair $\frac{m}{n}$ and $\frac{p}{q}$ of fractions with different denominators, one way (probably not the most convenient way) of replacing it with equivalent fractions that have the same denominator is to multiply the numerator and denominator of $\frac{m}{n}$ by p, (the denominator of $\frac{p}{q}$), and multiply the numerator and denominator of $\frac{p}{q}$ by n, (the denominator of $\frac{m}{n}$). The resulting fractions $\frac{mq}{nq}$ and $\frac{pn}{nq}$ have the same denominator np. To add them, proceed as above:

$$\frac{m}{n} + \frac{p}{q} = \frac{mq}{nq} + \frac{pn}{nq} = \frac{mq+np}{nq}$$

To subtract $\frac{p}{q}$ from $\frac{m}{n}$, do what you would expect:

$$\frac{m}{n} - \frac{p}{q} = \frac{mq}{nq} - \frac{pn}{nq} = \frac{mq-np}{nq}$$

Example 4

1. *To determine* $\frac{3}{8} + \frac{2}{5}$ *we first determine a common denominator. Look for a number that can be divided by 8 and also by 5. An example is 40. Now replace* $\frac{3}{8}$ *with* $\frac{3 \times 5}{8 \times 5}$ *and* $\frac{2}{5}$ *with* $\frac{2 \times 8}{5 \times 8}$. *Therefore*

$$\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{31}{40}$$

2. *To determine* $\frac{9}{8} - \frac{5}{6}$ *we first determine a common denominator. Look for a number that can be divided by 8 and also by 6. An example is 24. Now replace* $\frac{9}{8}$ *with* $\frac{9 \times 3}{8 \times 3}$ *and* $\frac{5}{6}$ *with* $\frac{5 \times 4}{6 \times 4}$. *Therefore*

$$\frac{9}{8} - \frac{5}{6} = \frac{27}{24} - \frac{20}{24} = \frac{7}{24}$$

Of course you may use $8 \times 6 = 48$ *as a common denominator. Then you would proceed as follows:*

$$\frac{9}{8} - \frac{5}{6} = \frac{9 \times 6}{8 \times 6} - \frac{5 \times 8}{6 \times 8} = \frac{54}{48} - \frac{40}{48} = \frac{14}{48} = \frac{14 \div 2}{48 \div 2} = \frac{7}{24}$$

3. $\frac{3}{4} - \frac{4}{5} = \frac{15}{20} - \frac{16}{20} = \frac{15 - 16}{20} = -\frac{1}{20}$

4. $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} = \frac{6}{12} - \frac{8}{12} + \frac{9}{12} = \frac{6 - 8 + 9}{12} = \frac{7}{12}$

Multiplying Rationals

Multiplying a fraction by a natural number n may be viewed as repeated addition. For example, $3 \times \frac{2}{15}$ may be viewed as

$$\frac{2}{15} + \frac{2}{15} + \frac{2}{15}$$

This is equal to

$$\frac{2+2+2}{15} = \frac{3 \times 2}{15} = \frac{6}{15}$$

Similarly, $6 \times \frac{2}{15}$ may be viewed as

$$\frac{2}{15} + \frac{2}{15} + \frac{2}{15} + \frac{2}{15} + \frac{2}{15} + \frac{2}{15} = \frac{6 \times 2}{15} = \frac{12}{15} = \frac{4}{5}.$$

In general, if m is an integer and $\frac{p}{q}$ is a rational number then their product is

$$m \times \frac{p}{q} = \frac{mp}{q}.$$

We may write m as $\frac{m}{1}$ and view $m \times \frac{p}{q}$ as $\frac{m}{1} \times \frac{p}{q} = \frac{m \times p}{1 \times q} = \frac{mp}{q}$. It turns out that any two fractions are multiplied in this same way. In other words, to multiply fractions $\frac{m}{n}$ and $\frac{p}{q}$, simply multiply their numerators and their denominators:

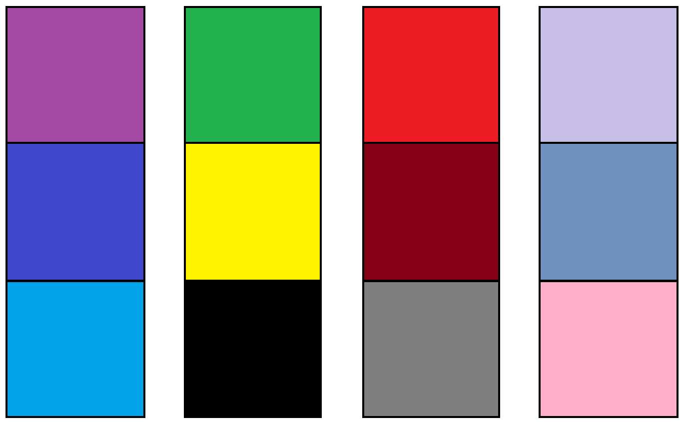
$$\frac{m}{n} \times \frac{p}{q} = \frac{mp}{nq}$$

Dividing Rationals

Dividing is repeated subtraction. For example, $2 \div \left(\frac{1}{4}\right)$ is 8 because if you set out to subtract $\frac{1}{4}$'s from 2 you will do so a total of 8 times and there will be no remainder. Alternatively, imagine that you have 2 dollars in quarters. If you decide to donate a quarter to each child that comes your way, you will donate to a total of 8 children.

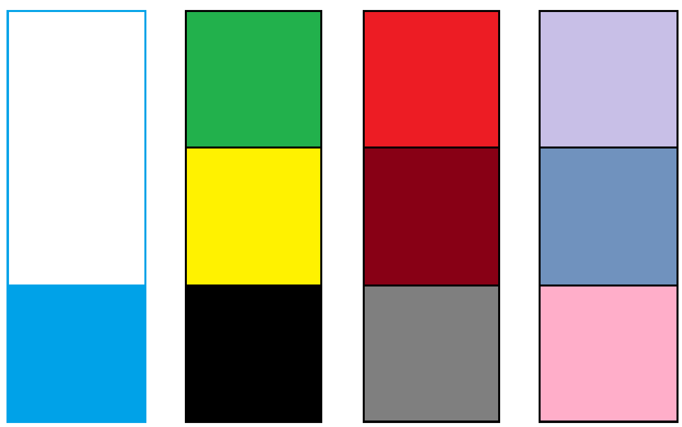
Similarly, $4 \div \left(\frac{2}{3}\right)$ is 6. One way to view this as a problem of repeated subtraction is to imagine that you have 4 ounces of a cough syrup and you must take $\frac{2}{3}$ of an ounce per day. Then dividing 4 by $\frac{2}{3}$ is equivalent to answering the question: "how many days will the syrup last?"

The four ounces are represented by 4 rectangles shown below. Since you take "thirds" of an ounce, (two of them), it is reasonable to divide each ounce into three equal parts, hence each color represents $\frac{1}{3}$ of an ounce.



You are required to take $\frac{2}{3}$ of an ounce a day, therefore:

- The first ounce will take you through the first day, with $\frac{1}{3}$ of an ounce to carry over to the next day.



- The $\frac{1}{3}$ of an ounce you carry over from the first day, (the blue part), combined with another ounce, (e.g. combined with the green, yellow and black parts) will take you through the next 2 days. Thus 2 ounces take you through the first 3 days. That leaves you with two ounces, which will take you through the next 3 days for a total of 6 days.

It turns out that the answer 6 above is also the result of multiplying 4 by $\left(\frac{3}{2}\right)$. I.e.

$$4 \div \frac{2}{3} = 4 \times \frac{3}{2} = \frac{4 \times 3}{2} = \frac{12}{2} = 6$$

Thus to divide 4 by $\frac{2}{3}$, simply multiply 4 by $\frac{3}{2}$, (called the reciprocal of $\frac{2}{3}$.)

Likewise, to divide 2 by $\frac{1}{4}$, simply multiply 2 by $\frac{4}{1}$ and the result is $2 \times \dfrac{4}{1} = \dfrac{2 \times 4}{1} = \dfrac{8}{1} = 8$.

In general, to divide by a fraction, multiply by the reciprocal of the fraction. Examples:

1. $9 \div \frac{3}{4} = 9 \times \frac{4}{3} = \frac{9 \times 4}{3} = \frac{36}{3} = 12$

2. $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = \frac{6}{5}$

3. $\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$

Percentages

Percentages are some of the common fractions we encounter. They are every where in test scores, in taxes, in interest rates, etc. By definition, a percentage is a fraction whose denominator is 100. Examples:

$$\frac{82}{100}, \quad \frac{112}{100}, \quad \frac{2.8}{100}, \quad \frac{\frac{1}{2}}{100}, \qquad \text{in general any fraction } \frac{p}{100} \text{ where } p \text{ is a given number.}$$

We write $\frac{p}{100}$ as $p\%$. Thus the other 4 examples above may be written as 82%, 112%, 2.8% and $\frac{1}{2}\%$ respectively.

Adding/subtracting percentages is easy since they have the same denominator 100. Thus

$$65\% + 22\% = \frac{65}{100} + \frac{22}{100} = \frac{87}{100} = 87\%$$

In fact it suffices to simply write $65\% + 22\% = 87\%$. Similarly,

$$28\% - 3\% = 25\%$$

We multiply or divide by percentages the same way we multiply or divide by arbitrary fractions.

Example 5 *The sales tax in a certain city is 8% of the listed price. (a) What is the sales tax on an item listed as costing $73.00 before tax? (b) What was the listed price, (i.e. the price before tax), of an item that was bought for $104.76 with the tax included?*

(a) The tax on a $73.00 item is $\frac{8}{100}$ *of* 73 *which equals* $\frac{73}{100} \times 8 = \frac{73 \times 8}{100} = 5.84$ *dollars*

(b) If the price of the item before tax was x *dollars then the price with the tax included was*

$$x + \frac{x}{100} \times 8 = x + \frac{8x}{100} = \frac{100x}{100} + \frac{8x}{100} = \frac{108x}{100}$$

Therefore

$$\frac{108x}{100} = 104.76.$$

To get x*, multiply both sides of the above equation by 100 then divide both sides by 108. The result is*

$$x = \frac{104.76 \times 100}{108} = 97$$

Thus the price of the item before tax was $97.00. Note that you ***do not*** *get the price before tax by subtracting 8% of $104.76 from $104.76. (If you do that you get $96.38, to the nearest cent, but an item with a list price of $96.38 costs $104.09 after tax.) The reason is that the sales tax IS NOT 8% of $104.76. It is 8% of the unknown list price.*

Exercise 6

1. *Give an equivalent fraction with the given denominator.*

   (a) $\frac{3}{8} = \frac{\quad}{32}$ *b)* $\frac{9}{17} = \frac{\quad}{119}$ *c)* $\frac{7}{8} = \frac{\quad}{100}$ *(d)* $7 = \frac{\quad}{5}$ *(e)* $\frac{3}{17} = \frac{\quad}{51}$ $\quad \frac{3}{2y} = \frac{\quad}{18y}$ $\quad \frac{2}{3} = \frac{\quad}{12n}$

2. *Calculate and simplify as much as possible.*

$$(a)\ \frac{2}{3}+\frac{1}{4} \quad (b)\ \frac{9}{14}-\frac{3}{8} \quad (c)\ \frac{5}{12}-\frac{3}{16} \quad (d)\ 29\%+\frac{4}{5} \quad (e)\ \frac{3}{7}+\frac{4}{21}-\frac{5}{6}$$

3. *Calculate and simplify as much as possible.*

$$(a)\ \frac{5}{6}\times\frac{1}{2} \quad b)\ \frac{5}{12}\times\frac{42}{65} \quad (c)\ \frac{15}{8}\times\frac{16}{3} \quad (d)\ \frac{19}{64}\times\frac{48}{95} \quad (e)\ \frac{3}{5}\times\frac{10}{11}$$

4. *A carpenter built a header by nailing a* $1\frac{5}{8}$ *- inch board to a* $3\frac{1}{2}$ *- inch beam. Find the thickness of the header.*

5. *At the beginning of the week, the Simplex Electric Company stock was selling at* $\$37\frac{3}{8}$ *per share. During the week, the stock gained* $\$5\frac{3}{4}$ *per share. Find the price of the share at the end of the week.*

6. *An electrician worked overtime as follows:* $2\frac{2}{3}$ *hours on Monday,* $1\frac{1}{4}$ *hours on Wednesday,* $1\frac{1}{3}$ *hours on Friday and* $6\frac{3}{4}$ *hours on Saturday.*

   (a) *Find the total number of overtime hours worked during the week.*

   (b) *At a salary of $22 an hour, how much overtime pay did the electrician receive?*

7. *During the first nine months of a year, there were* $12\frac{3}{8}$ *inches of rainfall.* $2\frac{3}{4}$ *inches fell during October,* $5\frac{1}{8}$ *inches during November, and* $4\frac{1}{3}$ *inches in December. Find the total rainfall for the year.*

8. *A 12 - mile race has 2 checkpoints. The first checkpoint is* $3\frac{3}{8}$ *miles from the starting point. The second check point is is* $4\frac{1}{3}$ *miles from the first checkpoint.*

   (a) *How many miles is it from the starting point to the second checkpoint?*

   (b) *How many miles is it from the second checkpoint to the finishing line?*

9. *A student can walk* $3\frac{1}{3}$ *miles in one hour. How many miles can he walk in* $2\frac{2}{5}$ *hours?*

10. *A compact car gets 28 miles to a gallon of gas. How many miles can it travel on* $5\frac{3}{4}$ *gallons?*

11. *1200 people attended a concert at a music center. The center was* $\frac{2}{3}$ *full. What is its full capacity?*

12. *A home building contractor bought* $4\frac{1}{2}$ *acres of land for $63,000. What was the cost per acre?*

13. *A developer purchased* $9\frac{3}{4}$ *acres of land for a building project. One and a half acres were set aside for a park*

    (a) *How many acres of land are available for building?*

    (b) *How many* $\frac{1}{4}$ *- acre parcels of developed land can be sold?*

14. *When one eats at a certain restaurant, the final bill will consist of (i) the cost of the meal, (ii)* 8% *of the cost of the meal as sales tax and (iii)* 15% *of the cost of the meal plus the sales tax, (i.e.* 15% *of the sum of the bills in (i) and (ii)) as a tip.*

    (a) *What is the final bill, to the nearest cent, for a meal costing $36?*

    (b) *What is the price, to the nearest cent, of a meal with a final bill of $74.52?*

15. *The dosage of a medicine is* $\frac{1}{4}$ *ounce for every 50 pounds of body weight. How many ounces of this medication are required for a person who weighs 175 pounds?*

16. *A biologist estimates that the human body contains 90 pounds of water for every 100 pounds of body weight. Estimate the number of pounds of water in a child weighing 75 pounds.*

17. *A survey indicates that 19 out of 30 eligible voters will vote in the next election. How many votes are estimated to be cast out of 528,000 eligible voters?*

18. *When a department store held a sell, the price of every item was reduced by* 60%. *If, in addition, a customer paid with the store's credit card, then the already reduced price would be reduced by another* 10%. *The final bill would be the over-all reduced price plus a* 7% *sales tax of the over-all reduced price. What did a customer pay, to the nearest cent, for an item that had a pre-sale price of* \$88.00 *assuming that he chose to pay with the store credit card?*

19. *The price of gas at a certain gas station was* $0.87\frac{9}{10}$ *dollars in June 1998. It is now* 3.64 *dollars. By how much has the price risen?*

20. *(A Brain Teaser.) It was wartime when Ricardos found out that Mrs. Ricardos was pregnant. Ricardos was drafted and made out a will, deciding that \$14,000 in a savings account was to be divided between wife and his child-to-be. Rather strangely, and certainly with gender bias, Ricardos stipulated that if the child were a boy, he would get twice the amount of the mother's portion. If it were a girl, the mother would get twice the amount the girl was to receive. We will never know what Ricardos was thinking of, for (as fate would have it) he did not return from war. Mrs. Ricardos gave birth to twins – a boy and a girl. How was the money divided?*

Manipulating Fractions With Variable Denominators or Numerators

We manipulate fractions with variable denominators or numerators the same way we manipulated fractions with specific numbers in the numerators/denominators. Thus;

- Before you add or subtract them, make sure they all have the same denominator then add or subtract the numerators
- To multiply them, simply multiply the numerators then multiply the denominators.
- To divide by a fraction, multiply by its reciprocal.

Example 7 *Assuming that the denominators of the given fractions are non-zero:*

1. $\frac{x}{3} - \frac{4}{5y} = \frac{5xy}{15y} - \frac{12}{15y} = \frac{5xy - 12}{15y}$
2. $\frac{2}{x+2} + \frac{4}{x-1} = \frac{2(x-1)}{(x+2)(x-1)} + \frac{4(x+2)}{(x+2)(x-1)} = \frac{6x+6}{(x+2)(x-1)}$
3. $\frac{1}{x} + \frac{x}{2} - \frac{1}{3} = \frac{6}{6x} + \frac{3x^2}{6x} - \frac{2x}{6x} = \frac{3x^2 - 2x + 6}{6x}$
4. $\frac{5}{x} \times \frac{(x+2)}{(x-5)} = \frac{5(x+2)}{x(x-5)} = \frac{5x+10}{x^2-5x}$.
5. $\frac{3}{5} \div \frac{x}{10} = \frac{3}{5} \times \frac{10}{x} = \frac{30}{5x} = \frac{6}{x}$

Exercise 8

1. *Combine into a single fraction:*

   (a) $\frac{2}{x} + \frac{3}{2x}$ (b) $\frac{x}{x+1} - \frac{2}{x}$ (c) $\frac{1}{x} + \frac{2}{3x} - \frac{4}{5x}$ (d) $\frac{1}{2x} - \frac{2x}{3}$ (e) $\frac{1}{x+1} - \frac{1}{x-1}$

2. *Perform the operation and simplify when possible*

   (a) *Multiply* $\frac{4}{7}$ *by* $\frac{3x}{2}$ (b) *Multiply* $\frac{x}{x+1}$ *by* $\frac{x+1}{x+3}$ (c) *Multiply* $\frac{2x}{3}$ *by* $\frac{3}{4}$

3. *What is* x *if*

   (a) $\frac{2}{x} = 6$? (b) $\frac{1}{x} + \frac{2}{3x} = \frac{3}{4}$?

Solutions to Practice Problems

1. Combine into a single fraction:

(a) $\frac{2}{3}+\frac{3}{5}$ (b) $\frac{3}{x}-\frac{2}{3x}$ (c) $\frac{1}{x}+\frac{2}{3x}-\frac{4}{5x}$ (d) $\frac{1}{2x}-\frac{2x}{3}$ (e) $\frac{1}{x+1}-\frac{1}{x-1}$

Solutions: (a) $\frac{2}{3}+\frac{3}{5}=\frac{2\times 5}{3\times 5}+\frac{3\times 3}{5\times 3}=\frac{10}{15}+\frac{9}{15}=\frac{10+9}{15}=\frac{19}{15}$

(b) $\frac{3}{x}-\frac{2}{3x}=\frac{3\times 3}{x\times 3}-\frac{2}{3x}=\frac{9}{3x}-\frac{2}{3x}=\frac{9-2}{3x}=\frac{7}{3x}$

(c) $\frac{1}{x}+\frac{2}{3x}-\frac{4}{5x}=\frac{1\times 15}{x\times 15}+\frac{2\times 5}{3x\times 5}-\frac{4\times 3}{5x\times 3}=\frac{15}{15x}+\frac{10}{15x}-\frac{12}{15x}=\frac{15+10-12}{15x}=\frac{13}{15x}$

(d) $\frac{1}{2x}-\frac{2x}{3}=\frac{1\times 3}{2x\times 3}-\frac{2x\times 2x}{3\times 2x}=\frac{3}{6x}-\frac{4x^2}{6x}=\frac{3-4x^2}{6x}$

(e) $\frac{1}{x+1}-\frac{1}{x-1}=\frac{1\times(x-1)}{(x+1)(x-1)}-\frac{1\times(x+1)}{(x-1)(x+1)}=\frac{(x-1)-(x+1)}{(x+1)(x-1)}=\frac{-2}{(x+1)(x-1)}$

2. A compact car gets 28 miles to a gallon of gas. How many miles can it travel on $5\frac{3}{4}$ gallons?

Solution: $5\frac{3}{4}$ gallons means 5 gallons plus $\frac{3}{4}$ of a gallon. This is a total of $\frac{20}{4}+\frac{3}{4}=\frac{23}{4}$ gallons. Since the car goes 28 miles on a gallon of gas, it can go $\frac{23}{4}\times 28$ miles on $5\frac{3}{4}$ gallons. This simplifies to $23\times 7=161$ miles. Alternatively, note that it goes $5\times 28=140$ miles on 5 gallons, and it goes 7 miles on $\frac{1}{4}$ of a gallon. Therefore it goes 21 miles on $\frac{3}{4}$ of a gallon, which implies that it can go $140+21=161$ miles on $5\frac{3}{4}$ gallons.

3. A student can walk $3\frac{1}{3}$ miles in one hour. How many miles can he walk in $2\frac{2}{5}$ hours?

Solution: $3\frac{1}{3}$ miles means $3+\frac{1}{3}$ miles. This is equal to $\frac{9}{3}+\frac{1}{3}=\frac{10}{3}$ miles. Also $2\frac{2}{5}$ hours may be written as $2+\frac{2}{5}=\frac{10}{5}+\frac{2}{5}=\frac{12}{5}$ hours. Therefore he can walk $\frac{10}{3}\times\frac{12}{5}$ miles in $2\frac{2}{5}$ hours. This is equal to $\frac{10\times 12}{3\times 5}=8$ miles

4. 1200 people attended a concert at a music center. The center was $\frac{2}{3}$ full. What is its full capacity?

Solution: Since $\frac{2}{3}$ is 1200, $\frac{1}{3}$ is 600 people. Therefore full capacity which is $\frac{3}{3}$ is $3\times 600=1800$ people.

5. A home building contractor bought $4\frac{1}{2}$ acres of land for \$63,000. What was the cost per acre?

Solution: $4\frac{1}{2}$ acres means $4+\frac{1}{2}$ acres, which we may write as $\frac{8}{2}+\frac{1}{2}=\frac{9}{2}$ acres. They cost \$63000. Therefore 1 acre cost $63000\div\frac{9}{2}=63000\times\frac{2}{9}=7000\times 2=14000$ dollars.

6. What is x if (a) $\frac{2}{x} = 6$? (b) $\frac{1}{x} + \frac{2}{3x} = \frac{3}{4}$?

   Solution: (a) Multiply both sides of the equation by x then divide both sides by 6. The result is $x = \frac{2}{6} = \frac{1}{3}$ (b) $\frac{1}{x} + \frac{2}{3x} = \frac{1 \times 3}{x \times 3} + \frac{2}{3x} = \frac{3}{3x} + \frac{2}{3x} = \frac{5}{3x}$. Therefore we must find x if $\frac{5}{3x} = \frac{3}{4}$. Multiply both sides of this equation by x then divide both sides by $\frac{3}{4}$. The result is $x = \frac{5 \times 4}{3 \times 3} = \frac{20}{9}$.

7. The dosage of a medicine is $\frac{1}{4}$ ounce for every 50 pounds of body weight. How many ounces of this medication are required for a person who weighs 175 pounds?

   Solution: We need the number of "50 pounds" in 175 pounds. They are $\frac{175}{50} = \frac{7 \times 25}{2 \times 25} = \frac{7}{2}$. the dosage should be $\frac{7}{2} \times \frac{1}{4} = \frac{7}{8}$ ounces.

8. When one eats at a certain restaurant, the final bill will consist of (i) the cost of the meal, (ii) 8% of the cost of the meal as sales tax, (iii) a tip equal to15% of the cost of the meal plus the sales tax. (a) What is the final bill, to the nearest cent, for a meal costing $36? (b) What is the price, to the nearest cent, of a meal with a final bill of $74.52?

   Solution: (a) The sales tax on $36 is $36 \times \frac{8}{100} = 2.88$ dollars. The cost of the meal plus tax is $36 + 2.88 = 38.88$ dollars. The tip on 38.88 dollars is $38.88 \times \frac{15}{100} = 5.83$ to the nearest dollar. Therefore the final bill is $38.88 + 5.83 = 44.71$ dollars to the nearest cent. (b) We want to know the cost of the meal before tax and tip. Let it be x dollars. Then the tax was $x \times \frac{8}{100} = 0.08x$ dollars. The cost of the meal plus tax was $x + 0.08x$ dollars. The tip on $1.08x$ dollars is $1.08x \times \frac{15}{100} = 0.162x$ dollars. Therefore the final bill was $1.08x + 0.162x = 1.242x$ dollars. This has to equal $74.52. Thus we have to find x given that $1.242x = 74.52$. the solution is $x = \frac{74.52}{1.242} = 60$. The meal cost $60.00

9. In some countries, (e.g. Canada), distances are measured in millimeters, centimeters, meters and kilometers, instead of inches, feet, yards and miles. Given that 5 miles are equivalent to 8 kilometers: (a) Convert 28 miles into kilometers. (b) Convert 89 kilometers into miles. (c) Convert x miles into kilometers. (d) Convert a speed of 55 miles per hour into kilometers per hour. (e) Convert a speed of 2 kilometers per minute into miles per hour.

   Solution: (a) If 5 miles are equivalent to 8 kilometers then i mile is equivalent to $\frac{8}{5}$ kilometers. Therefore 28 miles are equivalent to $\frac{8}{5} \times 28 = \frac{8 \times 28}{5} = 44.8$ kilometers. (b) 1 kilometer is equivalent to $\frac{5}{8}$ miles. Therefore 89 kilometers are equivalent to $\frac{5}{8} \times 89 = \frac{5 \times 89}{8} = 55.625$ miles. (c) x miles are equivalent to $\frac{8}{5} \times x = \frac{8x}{5}$ miles. (d) Travelling 55 miles in one hour is equivalent to travelling $\frac{8}{5} \times 55 = 88$ kilometers in one hour. Therefore a speed of 55 miles per hour is equivalent to a speed of 88 kilometers per hour. (e) If you maintain 2 miles per minute for an hour, you will cover $2 \times 60 = 120$ kilometers in one hour. This is equivalent to $\frac{5}{8} \times 120 = 75$ miles. Therefore a speed of 2 kilometers per minute is equivalent to a speed of 75 miles per hour.

10. A 12 - mile race has 2 checkpoints. The first checkpoint is $3\frac{3}{8}$ miles from the starting point. The second check point is is $4\frac{1}{3}$ miles from the first checkpoint. (a) How many miles is it from the starting point to the second checkpoint? (b) How many miles is it from the second checkpoint to the finishing line?

Solution: (a) It is $3\frac{3}{8} + 4\frac{1}{3}$ miles from the starting point to the second check point. Since $\dfrac{3}{8} + \dfrac{1}{3} = \dfrac{3 \times 3}{8 \times 3} + \dfrac{1 \times 8}{3 \times 8} = \dfrac{9+8}{24} = \dfrac{17}{24}$, the required distance is $7\frac{17}{24}$ miles. (b) It is $12 - 7\frac{17}{24}$ miles from the second check point to the finish line. To simplify, note that $12 - 7\frac{17}{24} = 5 - \frac{17}{24} = 4 + 1 - \frac{17}{24} = 4 + \frac{24}{24} - \frac{17}{24} = 4\frac{7}{24}$ miles.

11. A developer purchased $9\frac{3}{4}$ acres of land for a building project. One and a half acres were set aside for a park. (a) How many acres of land are available for building? (b) How many $\frac{1}{4}$ - acre parcels of developed land can be sold?

Solution: (a) There are $9\frac{3}{4} - 1\frac{1}{2}$ acres available for building. Since $\dfrac{3}{4} - \dfrac{1}{2} = \dfrac{3}{4} - \dfrac{2}{4} = \dfrac{1}{4}$, a total of $7\frac{1}{4}$ acres are available for building. The number of $\frac{1}{4}$ - acre parcels that may be sold is $(7\frac{1}{4}) \div \dfrac{1}{4} = \dfrac{29}{4} \div \dfrac{1}{4} = \dfrac{29}{4} \times \dfrac{4}{1} = 29$. Alternatively, simply note that there are 4 parcels in one acre, hence 28 in 7 acres. Since there are $7\frac{1}{4}$ acres, the total number of parcels is $28 + 1 = 29$.

12. When a department store held a sell, the price of every item was reduced by 60%. If, in addition, a customer paid with the store's credit card, then the already reduced price would be reduced by another 10%. The final bill would be the over-all reduced price plus a 7% sales tax of the over-all reduced price. What did a customer pay, to the nearest cent, for an item that had a pre-sale price of $88.00 assuming that he chose to pay with the store credit card?

Solution: The first 60% reduction cut the price to $88 - 88 \times \dfrac{60}{100} = 88 - 52.80 = 35.20$ dollars. He paid with the store credit card therefore he got an additional reduction of $35.20 \times \dfrac{10}{100} = 3.52$ dollars, to $35.20 - 3.52 = 31.68$ dollars. The sales tax on 31.68 dollars was $31.68 \times \dfrac{7}{100} = 2.22$ dollars to the nearest cent. Therefore he paid $31.68 + 2.22 = 33.90$ dollars.